

Boundary effects of electromagnetic vacuum fluctuations on charged particles

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Abstract. The effects of electromagnetic vacuum fluctuations with the boundary on charged particles is investigated. They may be observed via an electron interference experiment near the conducting plate, where boundary effects of vacuum fluctuations are found significant on coherence reduction of the electrons. The dynamics of the charge under the influence of quantized electromagnetic fields with a conducting plate is also studied. The corresponding stochastic equation of motion is derived in the semiclassical approximation, and the behavior of the charge's velocity fluctuations is discussed.

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1. Introduction

Manipulation of zero-point fluctuations due to the imposition of the boundary conditions may lead to an observable impact on macroscopic physics. One of the most celebrated examples is the attractive Casimir force between two parallel conducting plates [1]. Nevertheless, this induced-force effect can also be probed through the coupling to a test particle. Consider an atom in its ground state as a test particle located near a perfectly conducting plate. The boundary induced effects from electromagnetic vacuum fluctuations thus leads to a position-dependent energy shift, which further results in an attractive Casimir-Polder force on the atom toward the plate [2]. Thus, in this paper the motion of the test particle can serve as a probe to understand the nature of fluctuations from the effects on its dynamics. The dynamics of this particle and field interaction has been studied quantum-mechanically in the system-plus-environment approach [3, 4]. We treat the particle as the system of interest, and the degrees of freedom of fields as the environment. The influence of fields on the particle can be investigated with the method of Feynman-Vernon influence functional by integrating out field variables within the context of the closed-time-path formalism [5, 6]. The aim of this paper is to present a field-theoretic approach by considering the influence of fields with the boundary on the particle. The possible observational effects will be discussed.

2. Influence functional approach

We consider the dynamics of a nonrelativistic point particle of charge e interacting with quantized electromagnetic fields. In the Coulomb gauge, $\nabla \cdot \mathbf{A} = 0$, the Lagrangian is expressed as

$$L[\mathbf{q}, \mathbf{A}_T] = \frac{1}{2}m\dot{\mathbf{q}}^2 - V(\mathbf{q}) - \frac{1}{2} \int d^3\mathbf{x} d^3\mathbf{y} \varrho(x; \mathbf{q}) G(\mathbf{x}, \mathbf{y}) \varrho(y; \mathbf{q}) + \int d^3\mathbf{x} \left[\frac{1}{2}(\partial_\mu \mathbf{A}_T)^2 + \mathbf{j} \cdot \mathbf{A}_T \right], \quad (1)$$

in terms of the transverse components of the gauge potential \mathbf{A}_T , and the position \mathbf{q} of the point charge. The instantaneous Coulomb Green's function satisfies the Gauss's law, meanwhile the charge and current densities take the form

$$\varrho(x; \mathbf{q}(t)) = e \delta^{(3)}(\mathbf{x} - \mathbf{q}(t)), \quad \mathbf{j}(x; \mathbf{q}(t)) = e \dot{\mathbf{q}}(t) \delta^{(3)}(\mathbf{x} - \mathbf{q}(t)). \quad (2)$$

Let $\hat{\rho}(t)$ be the density matrix of the particle-field system, and then it evolves unitarily according to

$$\hat{\rho}(t_f) = U(t_f, t_i) \hat{\rho}(t_i) U^{-1}(t_f, t_i) \quad (3)$$

with $U(t_f, t_i)$ the unitary time-evolution operator. It is convenient to assume that the state of the particle-field at an initial time t_i is factorizable as $\hat{\rho}(t_i) = \hat{\rho}_e(t_i) \otimes \hat{\rho}_{\mathbf{A}_T}(t_i)$. The more sophisticated scheme of the density matrix involving initial correlations can be found in Ref. [7]. The gauge field at the time t_i is assumed in the vacuum state. Since the coupling between the electron and the fields is linear, the field variables can be

traced out exactly. Thus, this influence functional takes full account of the backreaction from electromagnetic fields. The physics becomes more transparent when we write the evolution of the reduced density matrix in the following form

$$\rho_r(\mathbf{q}_f, \tilde{\mathbf{q}}_f, t_f) = \int d^3\mathbf{q}_1 d^3\mathbf{q}_2 \mathcal{J}(\mathbf{q}_f, \tilde{\mathbf{q}}_f, t_f; \mathbf{q}_1, \mathbf{q}_2, t_i) \rho_e(\mathbf{q}_1, \mathbf{q}_2, t_i), \quad (4)$$

where the propagating function $\mathcal{J}(\mathbf{q}_f, \tilde{\mathbf{q}}_f, t_f; \mathbf{q}_1, \mathbf{q}_2, t_i)$ is

$$\mathcal{J} = \int_{\mathbf{q}_1}^{\mathbf{q}_f} \mathcal{D}\mathbf{q}^+ \int_{\mathbf{q}_2}^{\tilde{\mathbf{q}}_f} \mathcal{D}\mathbf{q}^- \exp \left[i \int_{t_i}^{t_f} dt \left(L_e[\mathbf{q}^+] - L_e[\mathbf{q}^-] \right) \right] \mathcal{F}[\mathbf{j}_T^+, \mathbf{j}_T^-], \quad (5)$$

and the electron Lagrangian $L_e[\mathbf{q}]$ is given by

$$L_e[\mathbf{q}] = \frac{1}{2} m \dot{\mathbf{q}}^2 - V(\mathbf{q}) - \frac{1}{2} \int d^3\mathbf{x} d^3\mathbf{y} \varrho(x; \mathbf{q}) G(\mathbf{x}, \mathbf{y}) \varrho(y; \mathbf{q}). \quad (6)$$

Here we introduce the influence functional $\mathcal{F}[\mathbf{j}_T^+, \mathbf{j}_T^-]$,

$$\begin{aligned} \mathcal{F}[\mathbf{j}^+, \mathbf{j}^-] = \exp \left\{ -\frac{1}{2} \int d^4x \int d^4x' \left[\right. \right. \\ \mathbf{j}^+_i(x; \mathbf{q}^+(t)) \langle \mathbf{A}_T^{+i}(x) \mathbf{A}_T^{+j}(x') \rangle \mathbf{j}^+_j(x'; \mathbf{q}^-(t')) \\ - \mathbf{j}^+_i(x; \mathbf{q}^+(t)) \langle \mathbf{A}_T^{+i}(x) \mathbf{A}_T^{-j}(x') \rangle \mathbf{j}^-_j(x'; \mathbf{q}^-(t')) \\ - \mathbf{j}^-_i(x; \mathbf{q}^-(t)) \langle \mathbf{A}_T^{-i}(x) \mathbf{A}_T^{+j}(x') \rangle \mathbf{j}^+_j(x'; \mathbf{q}^+(t')) \\ \left. \left. + \mathbf{j}^-_i(x; \mathbf{q}^-(t)) \langle \mathbf{A}_T^{-i}(x) \mathbf{A}_T^{-j}(x') \rangle \mathbf{j}^-_j(x'; \mathbf{q}^-(t')) \right] \right\}, \quad (7) \end{aligned}$$

which contains full information about the backreaction effects of quantized electromagnetic fields on the electron, and is a highly nonlocal object. The Green's functions of the vector potential are defined by

$$\begin{aligned} \langle \mathbf{A}_T^{+i}(x) \mathbf{A}_T^{+j}(x') \rangle &= \langle \mathbf{A}_T^i(x) \mathbf{A}_T^j(x') \rangle \theta(t - t') + \langle \mathbf{A}_T^j(x') \mathbf{A}_T^i(x) \rangle \theta(t' - t), \\ \langle \mathbf{A}_T^{-i}(x) \mathbf{A}_T^{-j}(x') \rangle &= \langle \mathbf{A}_T^j(x') \mathbf{A}_T^i(x) \rangle \theta(t - t') + \langle \mathbf{A}_T^i(x) \mathbf{A}_T^j(x') \rangle \theta(t' - t), \\ \langle \mathbf{A}_T^{+i}(x) \mathbf{A}_T^{-j}(x') \rangle &= \langle \mathbf{A}_T^j(x') \mathbf{A}_T^i(x) \rangle \equiv \text{Tr} \{ \rho_{\mathbf{A}_T} \mathbf{A}_T^j(x') \mathbf{A}_T^i(x) \}, \\ \langle \mathbf{A}_T^{-i}(x) \mathbf{A}_T^{+j}(x') \rangle &= \langle \mathbf{A}_T^i(x) \mathbf{A}_T^j(x') \rangle \equiv \text{Tr} \{ \rho_{\mathbf{A}_T} \mathbf{A}_T^i(x) \mathbf{A}_T^j(x') \}, \quad (8) \end{aligned}$$

and can be explicitly constructed. In particular, the retarded Green's function and Hadamard function of vector potentials are defined respectively by

$$G_R^{ij}(x - x') = i \theta(t - t') \langle [\mathbf{A}_T^i(x), \mathbf{A}_T^j(x')] \rangle, \quad (9)$$

$$G_H^{ij}(x - x') = \frac{1}{2} \langle \{ \mathbf{A}_T^i(x), \mathbf{A}_T^j(x') \} \rangle. \quad (10)$$

In the presence of the perfectly conducting plate, the tangential component of the electric field \mathbf{E} as well as the normal component of the magnetic field \mathbf{B} on the plate surface vanish. When the plate is placed at the $z = 0$ plane, the transverse vector potential \mathbf{A}_T in the $z > 0$ region is given by [8, 9],

$$\begin{aligned} \mathbf{A}_T(x) &= \int \frac{d^2\mathbf{k}_\parallel}{2\pi} \int_0^\infty \frac{dk_z}{(2\pi)^{1/2}} \frac{2}{\sqrt{2\omega}} \left\{ a_1(\mathbf{k}) \hat{\mathbf{k}}_\parallel \times \hat{\mathbf{z}} \sin k_z z \right. \\ &+ a_2(\mathbf{k}) \left[i \hat{\mathbf{k}}_\parallel \left(\frac{k_z}{\omega} \right) \sin k_z z - \hat{\mathbf{z}} \left(\frac{k_\parallel}{\omega} \right) \cos k_z z \right] \left. \right\} e^{i\mathbf{k}_\parallel \cdot \mathbf{x}_\parallel - i\omega t} + \text{H.C.}, \quad (11) \end{aligned}$$

where the circumflex identifies unit vectors. The position vector \mathbf{x} is decomposed into $\mathbf{x} = (\mathbf{x}_\parallel, z)$ where \mathbf{x}_\parallel is the components parallel to the plate. Similarly, the wave vector is expressed by $\mathbf{k} = (\mathbf{k}_\parallel, k_z)$ with $\omega^2 = k_\parallel^2 + k_z^2$. The creation and annihilation operators obey the typical commutation relations of the free fields.

3. Decoherence induced by vacuum fluctuations with the boundary

First, we study the decoherence dynamics of the electron coupled to quantized electromagnetic fields in the presence of the conducting plate. Let us now consider the initial electron state vector $|\Psi(t_i)\rangle$ to be a coherent superposition of two localized states $|\psi_1\rangle$ and $|\psi_2\rangle$ along worldlines \mathcal{C}_1 and \mathcal{C}_2 , respectively, after they leave the beam splitter at the moment t_i ,

$$|\Psi(t_i)\rangle = |\psi_1(t_i)\rangle + |\psi_2(t_i)\rangle. \quad (12)$$

As such, the leading effect of the decoherence can be obtained by evaluating the propagating function (5) along prescribed classical paths of the electrons. Thereby, the diagonal components of the reduced density matrix $\rho_r(\mathbf{q}_f, \mathbf{q}_f, t_f)$ now becomes

$$\begin{aligned} \rho_r(\mathbf{q}_f, \mathbf{q}_f, t_f) &= |\psi_1(\mathbf{q}_f, t_f)|^2 + |\psi_2(\mathbf{q}_f, t_f)|^2 \\ &\quad + 2 e^{\mathcal{W}[\bar{\mathbf{j}}^1, \bar{\mathbf{j}}^2]} \text{Re} \left\{ e^{i\Phi[\bar{\mathbf{j}}^1, \bar{\mathbf{j}}^2]} \psi_1(\mathbf{q}_f, t_f) \psi_2^*(\mathbf{q}_f, t_f) \right\}, \end{aligned} \quad (13)$$

where the \mathcal{W} and Φ functionals are found to be

$$\begin{aligned} \Phi[\bar{\mathbf{j}}^1, \bar{\mathbf{j}}^2] &= \frac{1}{2} \int d^4x \int d^4x' \left[\bar{\mathbf{j}}_i^1(x; \mathbf{q}^1) - \bar{\mathbf{j}}_i^2(x; \mathbf{q}^2) \right] \\ &\quad \times G_R^{ij}(x - x') \left[\bar{\mathbf{j}}_j^1(x'; \mathbf{q}^1) + \bar{\mathbf{j}}_j^2(x'; \mathbf{q}^2) \right], \\ \mathcal{W}[\bar{\mathbf{j}}^1, \bar{\mathbf{j}}^2] &= -\frac{1}{2} \int d^4x \int d^4x' \left[\bar{\mathbf{j}}_i^1(x; \mathbf{q}^1) - \bar{\mathbf{j}}_i^2(x; \mathbf{q}^2) \right] \\ &\quad \times G_H^{ij}(x - x') \left[\bar{\mathbf{j}}_j^1(x'; \mathbf{q}^1) - \bar{\mathbf{j}}_j^2(x'; \mathbf{q}^2) \right]. \end{aligned} \quad (14)$$

Here $\bar{\mathbf{j}}^{1,2}$ is the classical current along the respective paths, \mathcal{C}_1 and \mathcal{C}_2 to be specified later. The evolution of the electron states $\psi_{1,2}(\mathbf{q}_f, t_f)$ is governed by the Lagrangian L_e in Eq. (6) with backreaction effects neglected. Under the classical approximation with the prescribed electron's trajectory dictated by an external potential V , we find that the exponent of the modulus of the influence functional describes the extent of the amplitude change of interference contrast, and is determined by the Hadamard function of vector potentials. Its phase results in an overall shift for the interference pattern, and is related to the retarded Green's function.

The path plane on which the electrons travel can be either parallel or perpendicular to the plate. When the path plane is normal to the conducting plate, the electron worldlines are given by $\mathcal{C}_{1,2} = (t, v_x t, 0, z_0 \pm \zeta(t))$. We will choose a frame \mathfrak{S} which moves along the worldline $(t, v_x t, 0, z_0)$ and has the same orientation as the laboratory frame. In this frame, the electrons are seen to have sideways motion in the z direction only. Then the \mathcal{W}_\perp functional depends on the z - z component of the vector potential

Hadamard function. Under the dipole approximation, the corrections to the decoherence functional due to the presence of the conducting plate is expressed in terms of the ratio $\xi = z_0/T$ with z_0 being the effective distance of the electrons to the plate and $2T$ the effective flight time. Asymptotically, the ratio $\mathcal{W}_\perp/|\mathcal{W}_0|$ is given by

$$\frac{\mathcal{W}_\perp}{|\mathcal{W}_0|} = \begin{cases} -2 + \frac{8}{5}\xi^2 + \mathcal{O}(\xi^4), & \xi \rightarrow 0; \\ -1 - \frac{3}{16}\frac{1}{\xi^4} + \mathcal{O}(\frac{1}{\xi^6}), & \xi \rightarrow \infty, \end{cases} \quad (15)$$

Here the \mathcal{W}_0 functional denotes the contribution solely from unbounded Minkowski vacuum.

On the other hand, when the path plane lies parallel to the conducting plate, here the electron worldlines are given by $\mathcal{C}_{1,2} = (t, v_x t, \pm\zeta(t), z_0)$. The same reference frame \mathfrak{S} is chosen so that the electrons are seen to move in the y direction. Then, the y - y component of the vector potential Hadamard function becomes relevant to the \mathcal{W}_\parallel . Following the same approximation, asymptotically $\mathcal{W}_\parallel/|\mathcal{W}_0|$ is obtained as

$$\frac{\mathcal{W}_\parallel}{|\mathcal{W}_0|} = \begin{cases} -\frac{16}{5}\xi^2 + \frac{144}{35}\xi^4 + \mathcal{O}(\xi^6), & \xi \rightarrow 0; \\ -1 - \frac{3}{16}\frac{1}{\xi^4} + \mathcal{O}(\frac{1}{\xi^6}), & \xi \rightarrow \infty. \end{cases} \quad (16)$$

It is found that the effects of coherence reduction of the electrons by zero-point fluctuations with the boundary are strikingly deviated from that without the boundary. Thus, the presence of the conducting plate anisotropically modifies electromagnetic vacuum fluctuations that in turn influence the decoherence dynamics of the electrons. Electron coherence is enhanced when the path plane of the electrons is parallel to the plate. This results from the suppression of zero-point fluctuations due to the boundary condition in the direction parallel to the plate. On the other hand, the electron coherence is reduced in the perpendicular configuration where zero-point fluctuations are boosted along the direction normal to the plate.

4. Stochastic dynamics of a point charge driven by electromagnetic vacuum fluctuations in the presence of the conducting plate

The anisotropy of electromagnetic vacuum fluctuations in the presence of the conducting plate has been studied via an interference experiment of the electrons, and is manifested in the form of the amplitude change and phase shift of the interference fringes [10, 11]. Here we wish to further explore the anisotropic nature of electromagnetic vacuum fluctuations by the motion of the charged particle [12, 13]. We assume that the particle initially is in a localized state, and thus its density matrix can be expanded by the position eigenstate of the eigenvalue \mathbf{q}_i ,

$$\hat{\rho}_e(t_i) = |\mathbf{q}_i, t_i\rangle \langle \mathbf{q}_i, t_i|. \quad (17)$$

In Eq. (7), it is found more convenient to change the variables \mathbf{q}^+ and \mathbf{q}^- to the average and relative coordinates, $\mathbf{q} = (\mathbf{q}^+ + \mathbf{q}^-)/2$ and $\mathbf{r} = \mathbf{q}^+ - \mathbf{q}^-$. Next, we introduce the auxiliary noise fields $\xi^i(t)$ with the Gaussian distribution function,

$$\mathcal{P}[\xi^i(t)] = \exp \left\{ -i \frac{\hbar}{2} \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \xi^i(t) G_H^{ij-1}[\mathbf{q}(t), \mathbf{q}(t'); t-t'] \xi^j(t') \right\} \quad (18)$$

and thus the imaginary part of the coarse-grained action can be expressed as a functional integration over $\xi_i(t)$ weighted by the distribution function $\mathcal{P}[\xi_i(t)]$. As a result, we end up with

$$\exp \left\{ \frac{i}{\hbar} S_{CG}[\mathbf{q}, \mathbf{r}] \right\} = \int \mathcal{D}\xi_i \mathcal{P}[\xi_i(t)] \exp \left\{ \frac{i}{\hbar} \left[\text{Re}\{S_{CG}[\mathbf{q}, \mathbf{r}]\} - \hbar e \int_{-\infty}^{\infty} dt r^i \left(\delta^{ij} \frac{d}{dt} - q^j(t) \nabla^i \right) \xi^j \right] \right\} \quad (19)$$

The expressions in the squared brackets on the right hand side is defined as the stochastic effective action, which consists of the real part of the coarse-grained effective action as well as a coupling term of the relative coordinate r^i with the stochastic noise ξ^i . The Langevin equation is obtained by extremizing the stochastic effective action and then setting r^i to zero. By doing so, we have ignored intrinsic quantum fluctuations of the particle, and that holds as long as the resolution of the measurement on length scales is greater than its position uncertainty. The Langevin equation is then given by [13]

$$\begin{aligned} m \ddot{\mathbf{q}}^i + \nabla^i V(\mathbf{q}(t)) + e^2 \nabla^i G[\mathbf{q}(t), \mathbf{q}(t)] \\ + e^2 \left(\delta^{il} \frac{d}{dt} - \dot{q}^l(t) \nabla_i \right) \int_{-\infty}^{\infty} dt' G_R^{lj}[\mathbf{q}(t), \mathbf{q}(t'); t-t'] \dot{q}^j(t') \\ = -\hbar e \left(\delta^{il} \frac{d}{dt} - \dot{q}^l(t) \nabla^i \right) \xi^l(t) \end{aligned} \quad (20)$$

with the noise-noise correlation functions,

$$\langle \xi^i(t) \rangle = 0, \quad \langle \xi^i(t) \xi^j(t') \rangle = \frac{1}{\hbar} G_H^{ij}[\mathbf{q}(t), \mathbf{q}(t'); t-t'] \quad (21)$$

This Langevin equation encompasses fluctuations and dissipation effects on the charge's motion from quantized electromagnetic fields via the kernels G_H^{ij} and G_R^{ij} respectively, which both are in turn linked by the fluctuation-dissipation relation [15].

The backreaction kernel function of electromagnetic fields appears purely classical due to the fact that the coupling between a point charge and electromagnetic gauge potentials is linear. The noise-noise correlation functions can in principle be computed by taking an appropriate statistical average with the distribution functional $\mathcal{P}[\xi^i(t)]$. The noise-averaged result that describes the dynamics of the mean trajectory reduces, in the case of free space, to the known Abraham-Lorentz-Dirac equation with the self-force [14]. In the presence of the boundary, it can be seen that radiation emitted by the charge in nonuniform motion should be bounced back and then impinge upon the charge at later times. This will give rise to an additional retardation effect, which in turn results in a non-Markovian evolution of the charged particle. The fluctuations off the mean trajectory are driven by the stochastic noise, and will be studied with backreaction

dissipation taken into consideration in a way that a underlying fluctuation-dissipation relation is obeyed. The noise-driven trajectory fluctuations thus are entirely of quantum origin as can be seen from an explicit \hbar dependence in the noise term as well as the noise-noise correlation. As it stands, this is a nonlinear Langevin equation with non-Markovian backreaction, and the noise depends in a complicated way on the charge's trajectory because the noise correlation function itself is a functional of the trajectory.

The integro-differential equation (20) can be cast into a form similar to the Lorentz equation [13]. We consider that a charged particle undergoes a harmonic motion, and assume that the amplitude of oscillation is sufficiently small. The appropriate approximation for a non-relativistic motion will be the dipole approximation. This approximation amounts to considering the backreaction solely from electric fields, and linearizing the Langevin equation in such a way that the equation of motion remains non-Markovian. The ultraviolet divergence arises due to the integration of backreaction effects from the free-space contribution over all energy scales of fields in the coincidence limit. The energy cutoff Λ is then introduced to regularize the integral. The cutoff scale can be chosen to be the inverse of the width of the electron wavefunction. It essentially quantifies the intrinsic uncertainty on the charged particle. The divergence is then absorbed into mass renormalization.

The velocity fluctuations of the charged oscillator near the boundary are found to grow linearly with time in the early stage of the evolution, and then are asymptotically saturated as a result of the fluctuation-dissipation relation [13]. From dimensional consideration, the saturated value of velocity fluctuations induced by the presence of the boundary is given by $\Delta v_B^2 \sim e^2/m_e^2 z_0^2$, while that arising from the electron's motion can be argued to be $\Delta v_M^2 \sim \bar{\omega}_0/m_e$. Then, the ratio of two effects is

$$\frac{\Delta v_M^2}{\Delta v_B^2} \sim 10^7 \left(\frac{\bar{\omega}_0}{10^{12} \text{ s}^{-1}} \right) \left(\frac{z_0}{\mu\text{m}} \right)^2. \quad (22)$$

As a result, velocity fluctuations owing to the electron's motion are overwhelmingly dominant if its distance to the plate z_0 is about the order of μm , and the oscillation frequency, say 10^{12} s^{-1} , is chosen below the plasma frequency of the plate 10^{16} s^{-1} [16]. The corresponding modification in the effective temperature is

$$T_{eff} \sim \frac{\hbar \bar{\omega}_0}{k_B} \sim 10 \left(\frac{\bar{\omega}_0}{10^{12} \text{ s}^{-1}} \right) \text{ K}, \quad (23)$$

where k_B is the Boltzmann constant.

5. Concluding remarks

The nature of electromagnetic vacuum fluctuations in the presence of the conducting plate is studied by its effects on charged particles. The aim of this paper is to present a field-theoretic approach by considering the influence of fields with the boundary on the particle with the method of Feynman-Vernon influence functional. The extension of the above study by involving thermal fluctuations within the same formalism is in progress.

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